MATHEMATICAL GAMES

Dr. Matrix, like Mr. Holmes, comes to an untimely and mysterious end

by Martin Gardner

The sons of the prophet are brave
men and bold,
And quite unaccustomed to fear,
But the bravest by far in the ranks
of the Shah
Was Abdul Abulbul Amir.
—ANONYMOUS BALLAD

Going through my files on Dr. Irving Joshua Matrix, the greatest numerologist in the world, I find notes on many escapades that I have not yet written about in his peripatetic career. There was the year he spent in Tübingen as founder and director of the Institute of General Eclectics, a philosophical school maintaining that all metaphysical and religious systems are in substantial agreement. (Hans Küng taught there for several months.) And I have never told about Dr. Matrix' revival in Bombay of phrenology, which he cleverly combined with the ancient Hindu technique of acupuncture (a method quite different from that of the Chinese). Nor have I disclosed details about his notorious Parisian brothel for dogs and cats, where the madam was a large red-haired chow from Hong Kong, and pets were given free numerological readings on Saturdays.

Perhaps someday I shall recount these odd episodes, but this month I must with a heavy heart speak of my visit with the wily old charlatan last April in Istanbul. I had been in Budapest attending an international magic convention at the Duna Inter-Continental Hotel, where I had a comfortable room with a breathtaking view of the Danube. Dr. Matrix' half-Japanese daughter Iva somehow learned I was there, and one day while I was out she telephoned, leaving the cryptic message "Jeremiah 33 : 3," followed by an Istanbul phone number.

The Gideon Bible in my room provided the verse: "Call unto me, and I will answer thee, and shew thee great and mighty things, which thou knowest not." Iva answered the phone when I called. Had I ever, she wanted to know, been to Istanbul? I told her I had not. She and her father would be there for a week, she said, staying at the Hilton Hotel on Taksim Square, on the European side of the ancient city, and they would be glad of my company.

I flew to Istanbul early the next morning, taking a room at the Santral Hotel, which is near the Hilton but considerably cheaper. When Iva came by for me at half-past ten, driving a rented American car, I was surprised to find her clad in a traditional burka of bright orange that covered everything except her hands and feet and her dark, enigmatic eyes. I was to call her Fatima, she said. Her father was in Istanbul on a top-secret mission for the U.S. Government, the nature of which she could not disclose. He had assumed the identity of a Muslim from Teheran and was using the name Abdul Abulbul Amir. Because he would not be free to meet us until late that afternoon she suggested that we explore the city.

We drove south down Istaklal in bumper-to-bumper traffic to the accompaniment of wildly honking horns. Iva zigzagged adroitly through incomprehensible traffic lights, navigating by way of the old Jewish quarter, past the cone-capped Galata Tower and across the Galata Bridge. The murky water on each side—the Bosporus to the east and the Golden Horn inlet to the west—heaved with flotsam. The stench of sewage diminished only slightly as we moved deeper into Istanbul's Asian sector and parked near the Great Bazaar.

What bedlam! The rubbishy streets lined with tiny shops throbbed and jangled with swarms of people in every imaginable mode of dress. The more traditional of the women wore long coats and head scarfs, but some were in smart European clothes and a few even wore shorts. All of them stared at Iva in her burka as if she had been transported by time machine from the days of Ali Baba. As we pushed our way through the crowds scrawny cats darted between our legs, and it seemed that everywhere we turned there were young boys either shining shoes on luridly decorated boxes or hawking black-market Marlboro cigarettes with shouts of "Mah-buh-ro." A strong scent of spices almost masked the smells wafting from the surrounding waters.

Iva paused at a table of costume jewelry and after lengthy haggling bought four inexpensive trinkets at four different prices. One item, a pair of scarlet earrings, cost $1 in U.S. currency. When the young shopkeeper, pretending to be angry at the low settlement, added the four prices on his pocket calculator, I noticed that he hit the multiplication button three times instead of the addition button. When I pointed this out in a whisper to Iva, she nodded but gave the man the $6.75 that showed on the calculator's display.

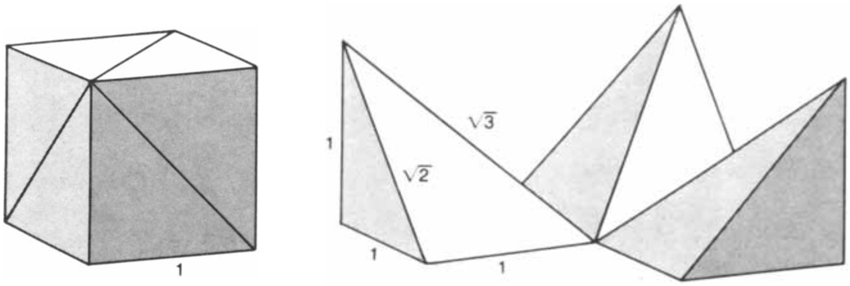
"Why didn't you protest?" I asked as we elbowed our way to another shop.

"Because," she replied, "I added the prices in my head and they came to the same amount."

I did some scribbling on the back of an envelope. "By the beard of the prophet!" I said. "You're right!"

Even more surprising, I later discovered that only one set of four different prices that includes $1 has $6.75 as both its product and its sum. Next month I shall give the solution to this pleasant little problem in Diophantine analysis.

We lunched at the Havuzlu restaurant, near the post office, and for the next four hours Iva took me around the city. We visited the Blue Mosque and the Topkapi Palace. We drove past the old Byzantine walls west of the city. It was distressing to see how many of the beautiful mosques are decaying. Some are used now for storing soft drinks; others house squatters. Once-elegant mosaic walls are pockmarked with gaps where the tiles have fallen. Even the domes and spires are stained brown



Dr. Matrix' cube (left) *cut to form three identical skew pyramids* (right) from pollution, and it was difficult to see them through the thick daytime haze.

When we finally arrived at the Hilton, Dr. Matrix was in his suite waiting for us, wearing a striped blue suit with a small emerald crescent in his lapel. His hair was closely cropped. I assumed that his gray beard and mustache were authentic, but his piercing green eyes had been turned black by contact lenses.

"You've not been in Afghanistan, I perceive," he said as we shook hands.

"No, thank Allah," I said with a smile, recognizing Dr. Matrix' parody of Sherlock Holmes's first remark to Watson. "How on earth did you know that?"

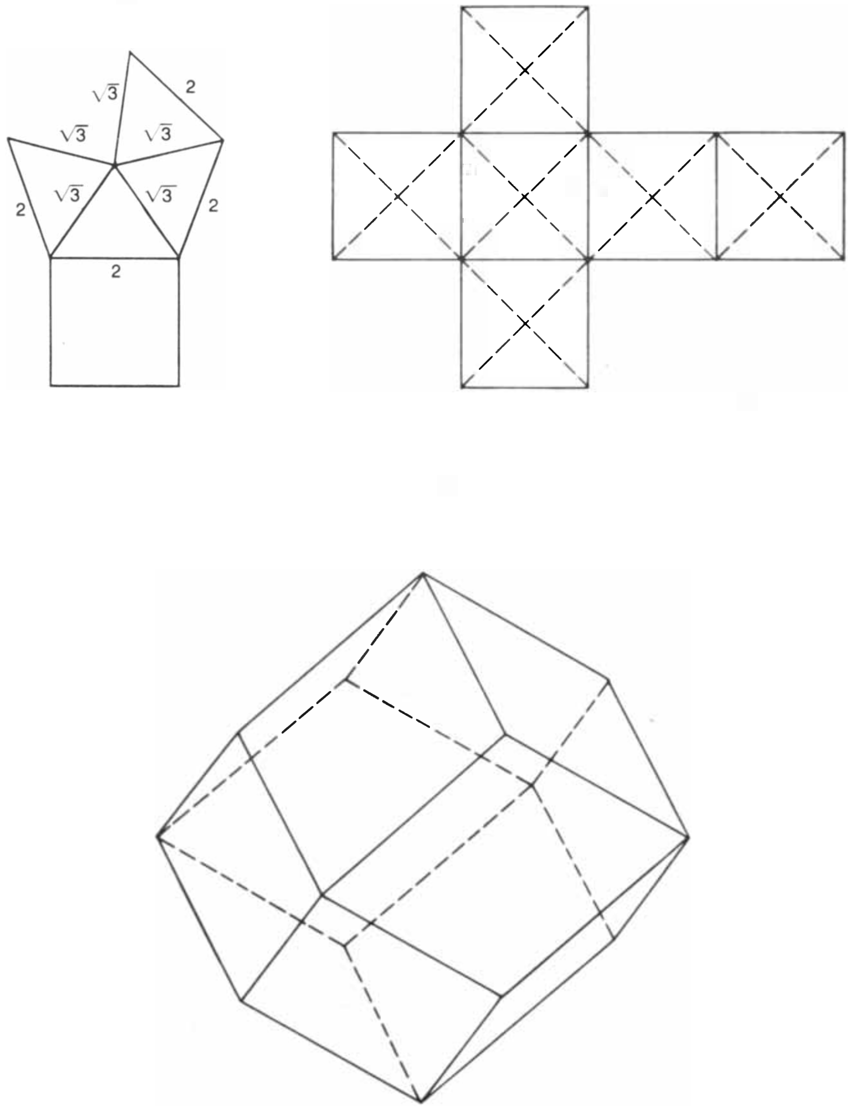
Dr. Matrix shrugged. "My daughter keeps good track of you."

Iva excused herself to change to less cumbersome attire, and Dr. Matrix and I seated ourselves in the bedroom he was also using as an office. On his desk was a large ivory cube that had been sliced in two places and hinged so that it opened to form three four-sided skew pyramids, each with a square base (as is shown in the illustration on the opposite page).

"The three pyramids are congruent," said Dr. Matrix. "If the square base has a side of 1, two adjacent faces are isosceles right triangles with sides of 1 and a hypotenuse that is $\sqrt{2}$. The other two sides are scalene right triangles with sides of 1 and $\sqrt{2}$ and a hypotenuse that is $\sqrt{3}$. The pyramids are easy to make with cardboard, but you would be surprised how many people have trouble putting them together to form a cube. The dissection goes back to ancient China. The pyramids were called yangmas. You might ask your readers if they can discover a completely different way to cut a cube into three identical solid forms."

Dr. Matrix picked up the hinged yangmas and folded them back until their square bases were mutually perpendicular. "Fit eight of these triplets over the eight corners of a cube of side 2," he continued, "and you create a rhombic dodecahedron. This construction provides an easy way to calculate the volume of such a solid. If the central cube has a side of 2, the rhombic dodecahedron has a volume of 8 + (24/3), or 16. Moreover, if you make four identical yangmas, they will fit together to form a pyramid that resembles the Great Pyramid of Egypt, with a 2-by-2 square base and four congruent isosceles triangles as sides."

The skeleton of a rhombic dodecahedron with its 12 identical diamond faces is shown at the bottom of the illustration on this page. The unfolded pyramid that can be made with four yangmas is shown at the top left in the illustration, and a fascinating toy can be created by gluing six of these pyramids at their bases to six square cells marked on a cross made of tape as is shown at the top right. Paint the bottom of the tape red and the sides of the pyramids blue.



Plans **(top)** *for a toy that forms both a rhombic dodecahedron* **(bottom)** *and a cube*

Folding the pyramids inward creates a solid red cube. Folding them outward, on the other hand, creates a blue rhombic dodecahedron with a cubic interior hole. With two such models it is possible to display a blue rhombic dodecahedron, remove its "shell" to disclose an interior red cube and fold the shell to make another red cube of the same size. Each cube can then be opened into two identical blue rhombic dodecahedrons.

Each corner of Dr. Matrix' ivory cube was marked with a different digit from the set 0, 1, 2, 3, 4, 5, 6, 7. The digits were cleverly placed, he told me, so that the sum of the two digits at the ends of each edge would be a prime number. (The primes are not necessarily different.) Can the reader find the only arrangement of the eight digits with this property before I give it next month?

"By the way," said Dr. Matrix as I jotted down the cube's number pattern, "are you aware that every cube has a volume equal to its surface area?"

He seemed mildly amused by the astonishment on my face. "Take any cube," he said, "and divide its edge into six equal parts. Call each part a hexling. A face obviously has an area of 36 square hexlings, and so since there are six faces, the total area is 6 × 36, or 216, square hexlings. The volume is of course 6^3, or 216, cubic hexlings. In the same way you can show that the area of any square is equal to its perimeter by dividing the side of the square into four equal quadlings. The paradox is related to a confusing proof that the surface-to-volume ratios of a sphere and a cube are the same."

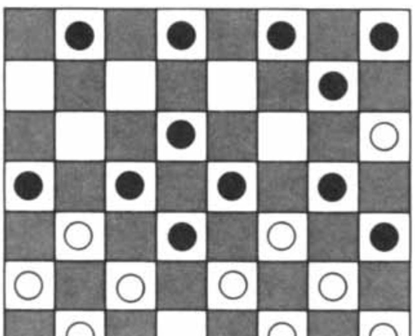
"But isn't it well known that of all solids the sphere has the smallest such ratio? That's the reason soap bubbles are spherical."

"That's true," said Dr. Matrix, "but hear me out." He explained the "proof" as follows. If d is the diameter of a sphere, its surface is πd^2 and its volume is $\frac{1}{6}\pi d^3$. The surface-to-volume ratio, then, reduces to $6/d$. Now let d be the edge of a cube. Here the surface-to-volume ratio is $6d^2/d^3$, which also reduces to $6/d$. Obviously something is wrong, but what is it?

"Enough of geometry," I said, my head spinning. "Have you encountered any number oddities since you came to Istanbul?"

Instead of replying, Dr. Matrix tossed over to me a 60-page booklet in English titled *Number 19: A Numerical Miracle in the Koran.* I later discovered that the author of this monograph, Rashad Khalifa, is an Egyptian who received a doctorate in biochemistry from an American university, where he also taught for a time. His booklet was published privately in the U.S. in 1972.

The number 19, Dr. Matrix pointed out, is as inscrutable to Muslims as 666, the Number of the Beast, is to Christians. Verses 27 through 31 of Sura 74 in the Koran tell how hell is guarded by 19 angels and explain that this number is intended to be an enigma for unbelievers. Dr. Khalifa's monograph attempts to show that 19 appears throughout the Koran too often to be there by chance. The number of suras in the Koran is 114, a multiple of 19. A famous verse called the *Basmala* ("In the name of Allah, most gracious, most merciful"), which opens every sura but one, has 19 letters. Its first word (*ism*) appears 19 times in the Koran. The second word (*Allah*) is found 2,698, or 142 × 19, times. The number of times the third word (*al-Rahman*) appears is 57, which is also a multiple of 19, as is the number of times the fourth word (*al-Raheem*) appears, 114.



FINAL POSITION

	RED	WHITE
1.	12-16	22-17
2.	16-20	23-19
3.	11-15	19-16
4.	9-14	16-12
5.	14-18	26-22
6.	5-9	31-26
7.	9-14	26-23
8.	6-9	23-19
9.	9-13	30-26
10.	7-11	26-23
11.	11-16	

The shortest no-capture checkers game known

"It's an ingenious study of the Koran," said Dr. Matrix, "but it could have been more impressive if Khalifa had consulted me before he wrote it. Nineteen is an unusual prime. For example, it's the sum of the first powers of 9 and 10 and the difference between the second powers of 9 and 10. Do you know what an emirp is?"

I shook my head.

"Well, *emirp* is *prime* backward, and it's the name my friend Jeremiah P. Farrell uses for any prime that is not a palindrome but that yields a different prime when its digits are reversed. For example, the last emirpal year was 1949, and the next will be 3011. Unfortunately both dates include duplicate digits, and to a numerologist emirps in which no two digits are alike are far more interesting. I call those numbers no-rep emirps, and their sequence is 13, 17, 31, 37, 71, 73, 79, 97, 107.... No one knows if the set of no-rep emirps is infinite. In fact, no one knows if there is a highest emirp or a highest palindromic prime."

"Is there any connection between emirps and Istanbul?" I asked.

"I'm coming to that," Dr. Matrix said. "As you know, Istanbul was once the great city of Constantinople. Its name was changed to Istanbul in 1930. Note the 19 and the 30. Nineteen is the mysterious Koran prime, and 30 is the largest integer n with the property that every smaller integer relatively prime to n is itself a prime. As you know, two integers are relatively prime if they have no common divisors, and Paul Erdös recently showed that 70 is the largest integer such that any smaller integer relatively prime to it is either a prime or a power of a prime." (See "A Property of 70," by Paul Erdös, in *Mathematics Magazine,* Vol. 51, No. 4, pages 238–240; September, 1978.)

"But I digress too far," Dr. Matrix said. "The most important date in the history of Constantinople is of course 1453, the year the city was conquered by the Turks. Now, 1,453 is not merely an emirp but also a no-rep emirp. Observe too that its digits add up to 13, which is the smallest emirp."

"Have there been many no-rep-emirp years since 1453?"

"There have been 11. The last was 1879 and the next will be 3019," Dr. Matrix said. "My good friend Leslie E. Card is the world authority on emirps, which he calls reversible primes. It's easy to determine that there are four pairs of two-digit emirps and 13 pairs of three-digit emirps. Card tells me there are 102 four-digit pairs and 684 five-digit pairs. He has a computer listing of all the no-rep emirps under 10,000,000. The breakdown by pairs is four with two digits, 11 with three, 42 with four, 193 with five, 612 with six and 1,790 with seven."

Card also discovered, Dr. Matrix told me, that only one six-digit emirp is cyclic, in the sense that if the first digit in the number is shifted repeatedly to the other end, each of the resulting permutations is an emirp. This unique number is 193,939. In other words, if this number is written with its digits in a circle, one can begin at any digit and go around in either direction to get a six-digit prime. The only cyclic emirp with five digits is 11,939. There are no cyclic emirps with three, four or seven digits.

Card has entertained himself, Dr. Matrix said, by constructing emirp squares of digits with the property that every row, column and main diagonal is a different emirp. Thus a square of n-by-n digits would contain $4(n+1)$ distinct primes. There is no such square of order 2 or 3. Examples for orders 4 and 5 are as follows:

9	1	3	3
1	5	8	3
7	5	2	9
3	9	1	1

1	3	9	3	3
1	3	4	5	7
7	6	4	0	3
7	4	8	9	7
7	1	3	9	9

There are many other order-5 squares, but the order-4 square is truly extraordinary: ignoring rotations and reflections, it is the only possible square of this type.

Can similar squares be made with no-rep emirps? No, because all primes except 2 and 5 end in 1, 3, 7 or 9: only those four digits can border an emirp square, and so if the square is of an order higher than 4, no outside prime will be free of repetitions.

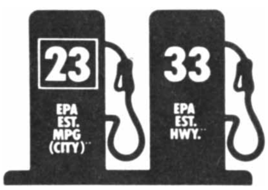
I wish I had space for more of Dr. Matrix' comments about primes. He also pointed out that the squares of the first seven primes add up to 666, and he mentioned the even more astounding fact that if the English names for primes are alphabetized, the first number on the list is 8,018,018,851. Is the last prime on this list also determinable? Dr. Matrix thought it was, but he suggested that a computer would be needed to find it.

At this point Iva, now dressed in gray silk pants and a yellow blouse, came in with a tray holding three martinis. We chatted about nonmathematical topics until the towers and domes of Istanbul became black silhouettes against a flaming gold-red sky. It was a vision straight out of *The Arabian Nights.* Sunsets, Iva pointed out, were the only admirable by-product of the city's dirty air. Through the open windows floated the wailing of a muezzin, his call to twilight prayer amplified by loudspeakers as he stood on a minaret not far away. (Mohammed disliked bells.) Dr. Matrix unrolled an intricately tessellated prayer rug and placed it on the floor with the point in its pattern directed southeast. After removing his shoes he recited the *Fatiha,* the Koran's first sura, in a loud voice and knelt on the rug and prostrated himself toward Mecca while Iva

PRESENTING AMERICA'S LOWEST-PRICED, BEST MILEAGE 4WD CARS. IN HATCHBACK. AND NO BACK.

The Subaru Hatchback and Brat® can both take quite a load off your back.

And both can really ease the burden on your monthly budget. To take home. And to feed.



Yet both include such nice extra touches as power assist front disc brakes, electronic ignition, and adjustable suspension for extra road clearance.

FRONT TO 4-WHEEL DRIVE AT THE FLICK OF A LEVER.

With either one, you don't have to stop just because the road does. Flick a lever, and you're in 4-wheel drive. Which is also great even when the road ahead is good, but the weather isn't. More traction means more peace of mind.

IMMEDIATE DELIVERY. NO WAITING.

You can start saving on the low sticker price and high mileage immediately. In fact, you can have one in days and hours. Not months and months.

Consider all the advantages: You get the practicality of 4WD for practically nothing. Plus the ability to take you over all the rough spots. At the end of the road. And the end of the month.

SUBARU.

INEXPENSIVE. AND BUILT TO STAY THAT WAY.

*Total suggested POE—not including dealer prep, inland transportation, state and local taxes, license and title fees, distributor or dealer installed options or accessories. California price is higher. Certain items shown optional at extra cost.

**Use estimated MPG for comparisons. Your mileage may differ depending on driving speed, weather conditions and trip length. Actual highway mileage will probably be less than highway estimate. Actual city mileage will be less in heavy traffic. California estimates lower.

sat sipping her martini with a bemused smile.

I spent several delightful days in Istanbul, and when I left, I fancied I could see tears in Dr. Matrix' eyes. Did he have a premonition about his kismet? His last words to me were "*Gulegule, Mashalla Hanim effendi*": "Good luck and Allah bless you."

"*Salaam,*" said Iva.

Three weeks later, back in New York, I was shattered by a story in *The New York Times.* It was datelined Bucharest. A Muslim known as Abdul Abulbul Amir, said to have been on a secret mission for the CIA, had met in Bucharest with a Russian agent, Ivan Skavinsky Skavar. The two had gone to a desolate spot on the delta of the Danube, outside the Ukrainian city of Izmail near the Romanian border. What happened there was unclear. Apparently the two men had fired revolvers simultaneously and both had died instantly. A peasant who witnessed the scene from a nearby hilltop reported hearing the taller man cry "*Allah Akbar!*" as he fell.

A few words may suffice to tell the little that remains. Amir's only surviving relative, a daughter named Fatima, had arranged for her father's burial in a tomb on the bank of the Danube near the spot where he had died. It was rumored, said the *Times,* that a group of Russians had taken Skavar's body aboard a ship and disposed of it in the Black Sea. No doubt I will learn more details if and when I see Iva again. With these sad words I close my final account of him whom I shall ever regard as the strangest and the wisest man I have ever known.

Here is how Ross Honsberger, who wrote last month's column, answers the three exercises given in his discussion of the pigeonhole principle:

1. To show that one of the line segments connecting five lattice points must pass through some lattice point in the coordinate plane, note that there are four "parity" classes for the coordinates of a lattice point: odd, odd; odd, even; even, odd, and even, even. On the pigeonhole principle some two of five lattice points, say (x_1, y_1) and (x_2, y_2), must belong to the same class. This implies that $x_1 + x_2$ and $y_1 + y_2$ are both even numbers, making the midpoint of the segment joining the points, namely $[(x_1 + x_2)/2, (y_1 + y_2)/2]$, a lattice point.

2. To prove that six circles arranged in the plane so that none of them contains the center of another cannot have a point in common, assume the converse is true, namely, that there is a point O common to six such circles. Now suppose O is joined to each of the six centers. No two centers can be colinear with O because no circle contains the center of another circle and all the circles contain O. Therefore the six lines all fan out from O. Let OA and OB be consecutive segments in the fan. Since O belongs to each circle, the segments OA and OB are not larger than the radii of the circles in which they lie. But since neither circle contains the other center, AB must be larger than either of these radii. Thus AB is longer than the other two sides of triangle AOB, which implies that angle AOB opposite AB is larger than either of the other angles in the triangle. Hence angle AOB must exceed 60 degrees. If this is so, however, there is not room in the 360-degree sweep around O for six angles such as AOB, which establishes the conclusion by contradiction.

3. To prove that in any row of $mn + 1$ distinct real numbers there is either an increasing sub-sequence of length $m + 1$ or a decreasing sub-sequence of length $n + 1$ let "coordinates" (x, y) be assigned as in example 7 given last month. The conclusion holds either if x is greater than m or if y is greater than n. Now, when x is less than or equal to m and y is less than or equal to n, there are only mn different pairs (x, y). On the pigeonhole principle two of the pairs assigned to the $mn + 1$ numbers in the row must be the same, and as was shown last month a contradiction follows.

Last June I reported that Alan Beckerson, a London expert on checkers problems, had found 28 final positions for a 24-move game in which no checkers are captured. For more than half a century the minimum number of moves for a no-capture game was thought to be 24, but that is not the case. Beckerson has now found several no-capture games that end after the 21st move. One such game is shown in the illustration on page 22.

For the material discussed this month I am indebted to many sources. The problem of the four prices is given in *Crux Mathematicorum* (Vol. 4, No. 4, pages 164–167; June, 1978). For the dissections of the cube see *Mathematical Models,* by H. Martyn Cundy and A. P. Rollett (Oxford University Press, second edition, 1961, page 122), and letters in *The Mathematical Gazette* (Vol. 57, No. 399, pages 66–67, February, 1973, and Vol. 57, No. 401, page 211, October, 1973). The problem of the numbers on the cube is from a personal letter from Garry Goodman, and the paradox of the cube's surface area and volume was sent in by Harlan L. Umansky.

Lam Garvin of the sheikhdom of Qatar called Khalifa's booklet to my attention by sending in clippings about it from this year's January 13 and January 20 issues of the weekly *Gulf Times,* which is published in Doha, the capital city. The 666 curiosity is a recent discovery made by Elvin J. Lee. The prime-alphabetization tasks are from Edward R. Wolpow's article "Alphabetizing the Integers," in *Word Ways* (Vol. 13, No. 1, pages 55–56; February, 1980).